Lecture 16

$\boxed{W \subset \mathbb{R}^n \ : \text{"}W \text{ is a set of vectors in } \mathbb{R}^n\text{"}}$

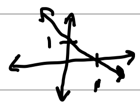
$\vec{v}_1, \ldots, \vec{v}_K \in \mathbb{R}^n$

$W = \text{span}(\vec{v}_1, \ldots, \vec{v}_K) = \{ \text{all linear combos of } \vec{v}_1, \ldots, \vec{v}_K \}$

How do you recognize when a given set $W \subset \mathbb{R}^n$ is a span?

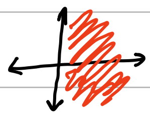
e.g $\mathbb{R}^n = \text{span}(\vec{e}_1, \ldots, \vec{e}_n)$

e.g. $W = \{ \binom{x_1}{x_2} \in \mathbb{R}^2 : x_1 + x_2 = 1 \}$



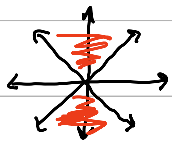
Not a span because $\vec{0} \notin W$

e.g. $W = \{ \binom{x_1}{x_2} \in \mathbb{R}^2 : x_1 \geq 0 \}$



Not a span because scaler times vectors are also in your span. You can multiply $\binom{1}{1}$ by -1 then the new vec is not in the domain.

e.g. $W = \{ \binom{x_1}{x_2} \in \mathbb{R}^2 : |x_1| \leq |x_2| \}$



Proposition : Given $\vec{v}_1, \ldots, \vec{v}_K \in \mathbb{R}^n$, let $W = \text{span}(\vec{v}_1, \ldots \vec{v}_K)$

Then :

(i) $\vec{0} \in W$

(ii) if $\vec{v} \in W$ and $c \in \mathbb{R}$, then $c\vec{v} \in W$

(iii) if $\vec{u}$ and $\vec{v} \in W$ then $\vec{u} + \vec{v} \in W$.

Def - $W \subset \mathbb{R}^n$ is a subspace if

(i) $\vec{0} \in W$

(ii) if $\vec{v} \in W$ and $c \in \mathbb{R}$, then $c\vec{v} \in W$

(iii) if $\vec{u}$ and $\vec{v} \in W$, then $\vec{u} + \vec{v} \in W$

Prop : $\forall\ \vec{v}_1, \ldots, \vec{v}_K \in \mathbb{R}^n$, span $(\vec{v}_1, \ldots, \vec{v}_K)$ is a subspace.

Examples of subspaces of $\mathbb{R}^n$

- $\mathbb{R}^n$
- <u>trivial</u> subspace $\{\vec{0}\} \subset \mathbb{R}^n$
- $T: \mathbb{R}^n \to \mathbb{R}^m$ linear transformation

  $\Rightarrow \ker(T) \subset \mathbb{R}^n$, $\text{image}(T) \subset \mathbb{R}^m$ are subspaces.

$$T(\vec{x}) = \underset{\substack{\parallel \\ [\vec{a}_1 \dots \vec{a}_n]}}{A}\vec{x}$$

$\text{image}(T) = \text{image}(A) = \text{span}(\vec{a}_1, \dots, \vec{a}_n)$

For $\ker(T)$, recall $\vec{x} \in \ker(T)$ i.f.f. $T(\vec{x}) = \vec{0}$

Since $T(\vec{0}) = \vec{0}$, we get $\vec{0} \in \ker(T)$.

Also $\vec{x} \in \ker(T)$, $c \in \mathbb{R} \Rightarrow T(c\vec{x}) = cT(\vec{x}) = c \cdot \vec{0} = \vec{0}$.

Finally if $\vec{x}, \vec{y} \in \ker(T)$ then $T(\vec{x}+\vec{y}) = T(\vec{x}) + T(\vec{y}) = \vec{0} + \vec{0} = \vec{0}$

So $\vec{x} + \vec{y} \in \ker(T)$.

<u>Defn</u>	Common Cold	Functions	Sets
Presceptive	Disease caused by family of viruses.	Matrix Transformation	Span of $\vec{v}_1, \dots, \vec{v}_k$
Descriptive	Symptoms (runny nose, sore throat).	Check if it is a Linear transformation	Subspaces

Question: Is every subspace of $\mathbb{R}^n$ a span?

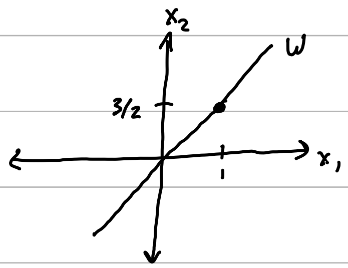
Defn: A **basis** for a subspace $W \subset \mathbb{R}^n$ is a linearly independent list of vectors $\vec{v}_1, \ldots, \vec{v}_k \in W$ that span W

Example: $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ is a basis for $\mathbb{R}^2$

So is $\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}$

$\{\vec{0}\}$ has no basis.

$\left\{ \begin{pmatrix}x_1\\x_2\end{pmatrix} \in \mathbb{R}^2 : 3x_1 = 2x_2 \right\}$



$\left\{ \begin{pmatrix}1\\3/2\end{pmatrix} \right\}$ is a basis for W.